Group members:

Warm-up: on your own, write down the three most challenging concepts you have learned since the first midterm. Then, as a group, share some strategies for those types of problems.

As a class: what are the important definitions, formulas, theorems, concepts, techniques, etc. that you should know for this exam?

Problem 1. (CW 4.5, Problem 5) For the function $g(x, y) = \sqrt{41 - 4x^2 - y^2}$, approximate $g(2.1, 2.9)$ using the point $(x_0, y_0) = (2, 3)$. How much error is involved in your approximation?

Problem 2. (CW 4.1 & 4.2, Problem 4) Draw the level curves $f(x, y) = k$ where $f(x, y) = \left(\frac{1}{2}y + x\right)^3$ and $k = -1, 0, 1, 8$.

Problem 3. Compute all first-order partial derivatives of the function $f(x, y) = x\sqrt{xy - 3}$.

Problem 4. Compute all first-order partial derivatives of the function $g(x, y) = \sin\left(\frac{1}{x+y}\right)$.

Problem 5. Compute all first-order partial derivatives of the function $h(x, y) = xy^2 e^{x+1}$.

Problem 6. What is the 12th order mixed partial derivative $f_{xxyyxyyyxxxy}$ for the function $f(x,y) = \sin x \cos y$?

Problem 7. (CW 4.3, Problem 4) Find a real number A that makes the function

$$f(x,y) = \begin{cases} \dfrac{x^2 - 2xy}{x^2 - 4y^2}, & x \neq \pm 2y \\ A, & (x,y) = (2,1) \end{cases}$$

continuous at $(x,y) = (2,1)$.

Problem 8. Compute the Taylor series of $\dfrac{e^{-x^2}}{x}$ centered about $x = 0$. What is the interval of convergence?

Problem 9. Express the definite integral

$$F(x) = \int_0^x \frac{t^2}{1-t^2}\, dt$$

as a Maclaurin series, using the Maclaurin series of $f(x) = \dfrac{x^2}{1-x^2}$. What is its interval convergence? Then use the fourth degree Taylor polynomial of $f(x)$ at $x = 0$ to approximate the integral

$$F\left(\tfrac{1}{2}\right) = \int_0^{1/2} \frac{t^2}{1-t^2}\, dt$$

Problem 10. Compute the radius of convergence and the interval of convergence for each of the power series.

(a) $\displaystyle\sum_{n=1}^{\infty}(n^3+n^2-1)x^n$

(b) $\displaystyle\sum_{n=1}^{\infty} n!\,x^n$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{(x+2)^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x)^n}{9^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{7^{n-1}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-4)^n}{(n+1)\,5^n}$

Problem 11. Consider the infinite series $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$.

(a) Use an appropriate series test to show that the series converges.

(b) Using part (a), can you decide the value of $\lim_{n\to\infty} \frac{\cos^2 n}{n^2}$? Explain.

Problem 12. (CW 3.3, Problem 9) For which value of c does $\sum_{n=0}^{\infty} e^{cn} = 10$?