Due Date: Wednesday, March 16 at 10AM EST

Carefully read and provide solutions to the problems below, showing all work required to justify any conclusions you make. You are encouraged to collaborate with your classmates, but all solutions turned in should be your own work. If you do collaborate, please record the names of those other students on your submitted work. Finally, your work should be submitted as a PDF on Canvas before the listed due date.

Textbook problems: Section 3.4 #12, 20; Section 3.5 #10, 14, 22, 24; Section 4.1 #4, 18, 40; Section 4.2 #8, 14, 30, 46

Optional textbook problems: the odd numbered problems from Sections 3.4 - 4.3.

Problem 1. Compute the Maclaurin series for the following function

$$F(x) = \int_0^x \frac{1}{4+t^2}\, dt.$$

What are its radius and interval of convergence? What is a more common name for $F(x)$? (For example, a 'more common name' for the function $\int_1^x \frac{1}{t}\, dt$ is $\ln(x)$.)

Problem 2. Let (a_n) be a sequence such that for all $n \geq 1$,

$$\frac{n + \sin\left(\frac{1}{n}\right)}{n} \leq a_n \leq \frac{n^4 + 3^n}{3^n - n}.$$

(a) Does the sequence (a_n) converge or diverge? If it converges, what does it converge to?

(b) Does the series $\sum_{n=1}^{\infty} a_n$ converge or diverge?

Problem 3. A circular plate of radius 5cm has centered identified with the point $(0,0)$ in the xy-plane. The temperature (in C°) on its surface is given by

$$T(x,y) = \sqrt{25 - x^2 - y^2}$$

for (x,y) such that $x^2 + y^2 \leq 25$. Sketch the isothermal curves (meaning, the curves of constant temperature) for $T = 5°\text{C}$, $4°\text{C}$, $0°\text{C}$, $-5°\text{C}$.

Problem 4. Let $f(x,y) = \ln(x+y)$.

(a) Sketch the domain of $f(x,y)$ as a region in the xy-plane.

(b) Draw the level curves $f(x,y) = k$ for $k = 0, 1, -1$. You may need the fact that $e \approx 2.7$ and $e^{-1} \approx 0.37$.