Group members:

Warm-up: describe three ways to compute a double integral. Be sure to carefully define your notation and state any hypotheses that are required.

Problem 1. Consider the domain $D = \{(x, y) \mid 0 \le x \le 5, -1 \le y \le 2\}$ and for a function $f(x, y)$ on D, approximate the volume under $z = f(x, y)$ by the following summation:

$$S = \sum_{i=1}^{15} f(x_i^*, y_i^*)\, \Delta A$$

where (x_i^*, y_i^*) are the midpoints of the 15 unit squares making up D and $\Delta A = 1$. For which functions below will S yield the *exact volume*?

$f(x, y) =$ $\quad x^2y \quad xy^2 \quad 5x^3 \quad \pi^2 \quad 5y - 5x \quad 25 \quad \pi^2 e^3 \quad \sin(x)\cos(y)$

Problem 2. Compute the volume under the graph $z = 2x - 4y^3$ over the rectangle

$$D = \{(x, y) \mid -5 \leq x \leq 4, 0 \leq y \leq 3\}.$$

Problem 3. Compute the double integral $\displaystyle\iint_D x \sec^2(y)\, dA$ over the rectangle

$$D = \left\{(x, y) \mid -2 \leq x \leq 3, 0 \leq y \leq \frac{\pi}{4}\right\}.$$

Problem 4. Compute the double integral $\iint_D xe^{xy}\,dA$ over the rectangle

$$D = \{(x, y) \mid -1 \leq x \leq 2, 0 \leq y \leq 1\}.$$

Hint: integrating in one order might be easier than integrating in the order.

Problem 5. Compute the double integral $\iint_D \frac{1}{(2x+3y)^2}\,dA$ over the rectangle

$$D = \{(x, y) \mid 0 \leq x \leq 1, 1 \leq y \leq 2\}.$$