Group members:

Warm-up: state the Lagrange multipliers theorem and explain what each part of the theorem means.

Problem 1. Let $f(x, y)$ be a differentiable function on a closed, bounded domain D in the xy-plane. Write down a strategy for identifying the absolute maximum and absolute minimum values of the function on this domain.

Problem 2. (Lecture 5.7, Q18) Consider the following two questions:

- Find the maximum value of $f(x,y)$ that satisfies $x^2+y^2 \le 9$.
- Find the maximum value of $f(x,y)$ that satisfies $x^2+y^2=9$.

(a) How are the questions different?

(b) Which question takes less work to solve? Explain how you know.

(c) Do solutions exist to both questions? What additional information would guarantee that they do?

Problem 3. (Lecture 5.7, Q10) Show that $(3,3)$ is not a local maximum of $f(x,y) = 2x^2-4xy+y^2-8x$ on the graph $x^3+y^3=6xy$.

Problem 4. Jim Bob is building a rectangular box with a square base and no top which has to have a volume of 54 cubic inches. Find the dimensions that minimize Jim Bob's expenses on the project, if the base material costs four times as much as the material for the sides.

Problem 5. Find the maximum value of $f(x, y, z) = xy^2z^3$ subject to the constraint $x + y + z = 1$, where x, y and z are all nonnegative.

Problem 6. Find the maximum volume of a rectangular box that can be inscribed in the ellipse $\frac{x^2}{4} + y^2 + z^2 = 1$.

Problem 7. Find the maximum value of the function $f(x, y, z) = x(y + z)$ given that $x^2 + y^2 = 1$ and $xz = 1$.