Group members:

Warm-up: state the Extreme Value Theorem for a multivariable function $f(x, y)$ and explain how you would check each of the conditions in the theorem.

Problem 1. (Lecture 5.6, Q32) Let $f(x, y)$ be a differentiable function and let

$$D = \{(x, y) : y \geq x^2 - 4, x \geq 0, y \leq 5\}.$$

(a) Sketch the domain D.

(b) Does the Extreme Value Theorem guarantee that f has an absolute minimum on D? Explain.

(c) List all the places you would need to check in order to locate the minimum.

Problem 2. Classify the critical point $(0,0)$ of $f(x,y) = \cos(2x+y) + xy$ as a local maximum, a local minimum or a saddle point.

Problem 3. Find and classify the critical points of $f(x,y) = (y-2)x^2 - y^2$.

Problem 4. Find the absolute minimum and maximum values of the function $f(x, y) = 2x^2 - y^2 + 6y$ on the region $x^2 + y^2 \leq 16$. *Hint: draw the region first.*

Problem 5. Find the absolute minimum and maximum values of the function $f(x, y) = 2x^3 - 4y^3 + 24xy$ on the region $0 \leq x \leq 5, -3 \leq y \leq -1$. *Hint: draw the region first.*

Problem 6. Find the absolute minimum and maximum values of the function $f(x,y) = 18x^2 + 4y^2 - y^2x - 2$ on the solid triangle with vertices $(-1,-1),(5,-1)$ and $(5,17)$. *Hint: draw the region first.*

Problem 7. After decades of research, the Lucky Tails Saddle Company has perfected their saddle design, which can be modeled by the graph of the function $h(x,y) = \frac{4}{5}x^2 - \frac{9}{10}y^2$ over the domain $R = \{(x,y) : -2 \leq x \leq 3, -2 \leq y \leq 2\}$.

(a) Find the saddle point of this saddle.

(b) A horse trainer informs Lucky Tails that in order for a saddle to fit most comfortably on a horse, its highest point should be in the direction of the horse's head. Using (x,y) coordinates, explain how to orient the saddle when placing it on a horse's back in order to fit most comfortably.