Group members:

Warm-up: write the chain rule for computing the derivative $\frac{df}{dt}$ of a function $f(x, y)$ where x and y are each functions of another variable t. Then discuss with your group how this compares to the chain rule for single variable functions.

Problem 1. Compute $\frac{df}{dt}$ where $f(x, y) = xe^{xy}$, $x(t) = t^2$ and $y(t) = \frac{1}{t}$.

Problem 2. Compute $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ where $z = e^{2x}\sin(3y)$, $x(s,t) = st - t^2$ and $y(s,t) = \sqrt{s^2+t^2}$.

Problem 3. Use the multivariable chain rule to compute $\frac{dy}{dx}$ where x and y satisfy the implicit equation $x\cos(3y) + x^3y^5 = 3x - e^{xy}$.

Problem 4. Compute $\frac{df}{dt}$ where $f(x,y,z) = \frac{x^2-z}{y^4}$, $x(t) = t^3+7$, $y(t) = \cos(2t)$ and $z(t) = 4t$.

Problem 5. A airplane is on approach to Hartsfield–Jackson Airport on an unknown trajectory, but its altitude h (in feet) is a function of its coordinates x and y on the 2-dimensional radar display in the control tower (both listed in miles). The plane is following a trajectory described by some vector valued function $\langle x(t), y(t)\rangle$ where t is time in minutes and the control tower staff are able to determine that right now, the plane's altitude and ground coordinates have the following rates of change:

$$\frac{\partial h}{\partial x} = -5 \text{ feet/mile}, \quad \frac{\partial h}{\partial y} = 2 \text{ feet/mile}, \quad \frac{dx}{dt} = 3 \text{ miles/minute} \quad \text{and} \quad \frac{dy}{dt} = 7 \text{ miles/minute}.$$

Find the current rate of change in the plane's altitude per minute.

Problem 6. If f is a function of 5 variables, say $f(x_1, x_2, x_3, x_4, x_5)$, and each x_i is a function of 3 variables, say $x_1(s,t,u), \ldots, x_5(s,t,u)$, how many terms will each of the partial derivatives $\frac{\partial f}{\partial s}$, $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial u}$ have? Write the formula for at least one of these partial derivatives.

Problem 7. Find the gradient ∇f of the function $f(x,y,z) = 3x^2 - 2xy + 4z^2$ if $x(s,t) = e^s \sin(t), y(s,t) = e^s \cos(t)$ and $z(s,t) = e^s$.

Problem 8. Jim Bob was meaning to practice his ice sculpting skills on a cylindrical block of ice, but he left the block out in the sun and it began to melt. When he found the block, it had a diameter of 2 feet which was decreasing at a rate of 0.4 feet per minute, and a height of 5 feet which was decreasing at a rate of 0.3 feet per minute. Calculate the rate at which the volume of the ice block was shrinking at that moment in time. If that rate of change in volume held, how long did it take for the block to complete melt?

Problem 9. Suppose $f(x,y) = xe^{xy^2}$, $x(t) = t^2$ and $y = 5t$. Determine the second derivative $\frac{d^2t}{dt^2}$ (which can also be written $f''(t)$ since f is a function of a single variable t).