Group members:

Warm-up: write a formula for the directional derivative of $f(x, y)$ in the direction of a unit vector $\vec{u} = \langle a, b \rangle$. What does this formula mean in words? Are the partial derivatives f_x and f_y examples of directional derivatives or are directional derivatives examples of partial derivatives?

Problem 1. Find $D_{\vec{u}} f(\pi, 0)$ where $f(x, y) = x\cos(y)$ and $\vec{u}$ is a unit vector in the direction of $\vec{v} = \langle 2, 1 \rangle$.

Problem 2. Find $D_{\vec{u}}f(2,0)$ where $f(x,y) = xe^{xy} + y$ and $\vec{u}$ is the unit vector in the xy-plane forming an angle of $\frac{2\pi}{3}$ radians with the positive x-axis.

Problem 3. Determine the gradient ∇f of each of the following functions:

(a) $f(x,y) = \sqrt{x^2 + y^4}$

(b) $f(x,y) = x^2 \sec(3x) - \frac{x^2}{y^3}$

Problem 4. Determine the maximum rate of change and the direction in which this rate of change occurs for the function $f(x, y) = \sqrt{x^2 + y^4}$ at the point $(-2, 3)$.

Problem 5. Suppose that the elevation of a hill above sea level is given by the height function $h(x, y) = 1000 - \frac{1}{100}x^2 - \frac{1}{50}y^2$. Standing at the point $(60, 100)$,

(a) in what direction is the elevation changing fastest?

(b) what is the greatest rate of change of elevation?

Problem 6. Use the gradient to find an equation for the tangent plane to the surface $z = x^3y - x^2y^2$ at the point $(1, 1, 0)$.

Problem 7. Identify possible peaks and valleys (maxima and minima) of the function $f(x, y) = -x^4 + 4x^2 - 4y^2 - 3$ by finding where the gradient ∇f is equal to $\langle 0, 0\rangle$. Can you tell which of these points are actually peaks and which are valleys?