Group members:

Warm-up: what does a vector represent? How do you know if two vectors $\vec{u}$ and $\vec{v}$ are the same? Describe some examples algebraically (with coordinates) and geometrically (with pictures).

Problem 1. Which of the following vectors are equal? Sort them into distinct groups based on if they are equal or not. (Such a sorting is called an "equivalence relation" and each group is an "equivalence class".)

$$\overrightarrow{AB},\quad \overrightarrow{AC},\quad \overrightarrow{BC},\quad \overrightarrow{BA},\quad \overrightarrow{CA},\quad \overrightarrow{CD},\quad \overrightarrow{BD},\quad \overrightarrow{BE},\quad \overrightarrow{DE},\quad \overrightarrow{ED},\quad \overrightarrow{AE},\quad \overrightarrow{CE}$$

where $A = (2, 1), B = (5, 3), C = (1, -1), D = (4, 1)$ and $E = (3, -1)$

Problem 2. For the vectors $\vec{u} = \langle -2, 1, 3\rangle, \vec{v} = \langle 2, 5, 1\rangle$ and $\vec{w} = \langle 4, -1, 2\rangle$, compute

(a) $\vec{u} + \vec{v}$

(b) $\vec{v} + 2\vec{w}$

(c) $3\vec{u} - 5\vec{w}$

(d) $\vec{u} + 2\vec{v} - \vec{w}$

(e) $\vec{u} \cdot \vec{v}$

(f) $\vec{u} \cdot \vec{w}$

(g) $(\vec{u} \cdot \vec{w})\vec{v}$

Problem 3. Compute the lengths of each of the vectors in Problem 2.

Problem 4. A *unit vector* is a vector with length 1. Find a unit vector that is in the same direction as each of the following vectors.

(a) $\langle 2, -1 \rangle$

(b) $\langle 1, -1 \rangle$

(c) $\langle -1, 0 \rangle$

(d) $\langle 3, 4 \rangle$

(e) $\langle 2, -1, 2 \rangle$

(f) $\langle -1, 0, 2 \rangle$

(g) $\langle 7, -1, 5 \rangle$

(h) $\langle -2, -1, 2 \rangle$

Problem 5. Find the coordinates of the point that is one fifth of the way from $A = (2, 1, -2)$ to $B = (1, -2, 0)$.

Problem 6. Decide if each statement is TRUE or FALSE. For the true statements, explain why. For the false statements, provide a counterexample illustrating that the statement is false. Assume all vectors are in 3 dimensional space unless stated otherwise.

(a) The zero vector $(0, 0, 0)$ is the only vector with length 0 in 3 dimensions.

(b) If $\vec{u}$ and $\vec{v}$ and $|\vec{u} - \vec{v}| = 0$, then $\vec{u} = \vec{v}$.

(c) If $|\vec{u}| = |\vec{v}|$ then $\vec{u} = \vec{v}$.

(d) If $\vec{u}$ and $\vec{v}$ are parallel, then they point in the same direction.

(e) If $\vec{u}$ and $\vec{v}$ point in the same direction, then they are parallel.

(f) $|\vec{u} + \vec{v}| = |\vec{u}| + |\vec{v}|$.

(g) If a is a real number, then $|a\vec{v}| = a|\vec{v}|$ for all vectors $\vec{v}$.

(h) If $\vec{u}$ and $\vec{v}$ are parallel and $\vec{v}$ and $\vec{w}$ are parallel, then $\vec{u}$ and $\vec{w}$ are parallel.

Problem 7. Find the angle between the vectors $\vec{v} = \langle 1, 3, 1 \rangle$ and $\vec{w} = \langle 3, 2, -9 \rangle$.

Problem 8. Find the distance between the points $A = (5, 1, -5)$ and $B = (11, 4, 1)$.

Problem 9. Let $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ be the equation for a plane in 3-dimensional space. Show that the vector $\vec{n} = \langle a, b, c \rangle$ is perpendicular to the plane, that is, $\vec{n}$ is perpendicular to any vector lying in the plane.