Group members:

Warm-up: describe in words what it means for the following to be true:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L.$$

What are some ways for the limit to not exist?

Problem 1. Consider the function $f(x,y) = \dfrac{xy}{x+y}$.

(a) What is the domain of $f(x,y)$?

(b) Compute the limit $\lim_{(x,y)\to(5,1)} f(x,y)$ or show that it does not exist.

Problem 2. Consider the function $f(x, y) = \frac{2x^2 - xy - y^2}{x^2 - y^2}$.

(a) What is the domain of $f(x, y)$?

(b) Compute the limit of $f(x, y)$ as (x, y) approaches $(1, 1)$ along the line $y = 1$. Pay attention to what technique you use to evaluate this limit.

(c) Compute the limit of $f(x, y)$ as (x, y) approaches $(1, 1)$.

Problem 3. Use the Squeeze Theorem to evaluate each of the following limits.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} x^2 \sin\left(\frac{1}{x^2+y^2}\right)$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{|xy|}{\sqrt{x^2+y^2}}$

Problem 4. Find a real number A that makes the function

$$f(x,y) = \begin{cases} \dfrac{x^2 - 2xy}{x^2 - 4y^2}, & x \neq \pm 2y \\ A, & (x,y) = (2,1) \end{cases}$$

continuous at $(x,y) = (2,1)$.

Problem 5. For which points (x,y) is the function

$$f(x,y) = \begin{cases} \dfrac{\cos(y)\sin(x)}{x}, & x \neq 0 \\ \cos(y), & x = 0 \end{cases}$$

continuous?