Group members:

Warm-up: write the formula for the Taylor series for $f(x)$ centered at $x = a$. What is the Maclaurin series for $f(x)$?

Problem 1. Use the definition of a Taylor series to find a power series representing the function $f(x) = e^{2x}$ centered at $x = 1$.

Problem 2. Use a known Taylor series to find a power series for $x\cos(3x^2)$ centered at $x = 0$.

Problem 3. Use power series to evaluate the limit $\lim_{x\to 0} \frac{\ln(1+x) - x}{x^2}$.

Problem 4. Use power series to compute the limits of the following infinite series. That is, show that they converge and find the value they converge to.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{3^n n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{2^n}$ Hint: Try differentiating a known power series.

Problem 5. If $f(x) = e^{x^2}$, find $f^{(50)}(0)$. Hint: try writing the Maclaurin series of $f(x)$.

Common Maclaurin series:

Series	Radius of convergence
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$	1
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x\frac{1}{2}x^2 + \frac{1}{6}x^3 + \ldots$	∞
$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{7!}x^7 + \ldots$	∞
$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \ldots$	∞
$\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \ldots$	1
$\ln(1+x) = \sum_{n=1} (-1)^{n-1} \frac{x^n}{n} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$	1