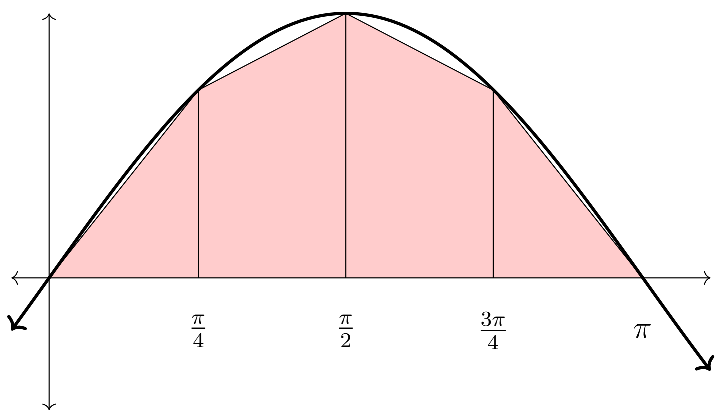
Group members:

Warm-up: write the formulas for L_n, R_n, M_n and T_n. What are some of the similarities and differences between these approximation formulas?

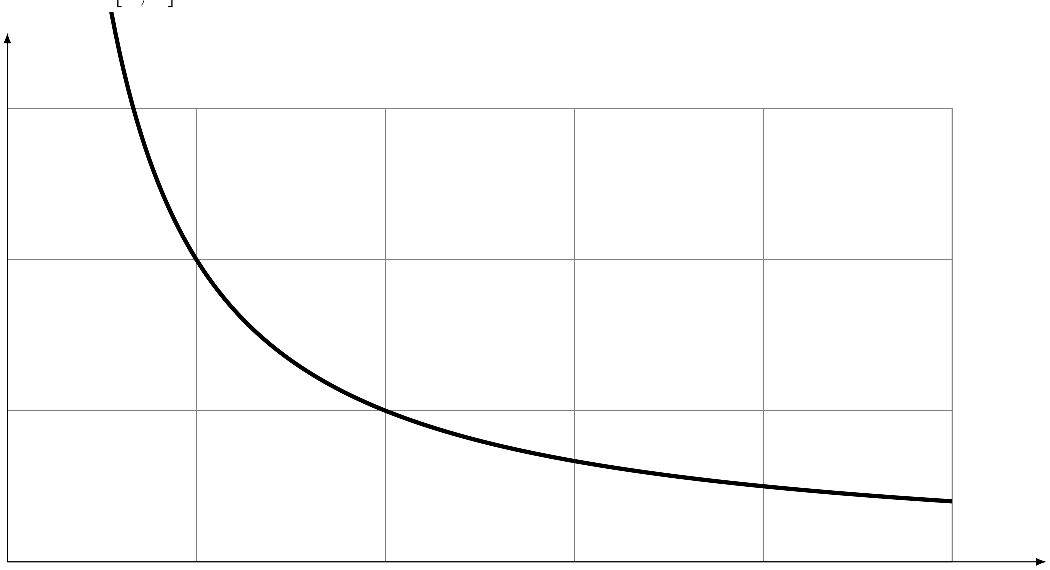
Problem 1. Let $f(x) = \sin x$ on the interval $[0, \pi]$. Let's divide the interval into $n = 4$ pieces and estimate the definite integral $\int_0^\pi \sin x \, dx$ using the Trapezoid Rule.



$$T_4 = \frac{\Delta x}{2}\big(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\big) =$$

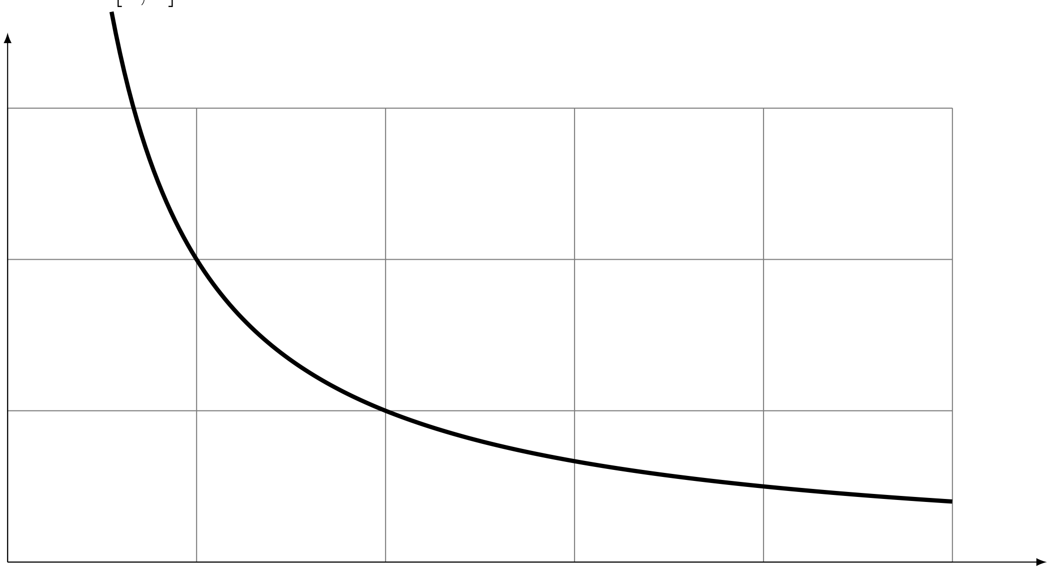
Compare this to the actual value of the definite integral:

Problem 2. On the graph below, draw the midpoint approximation M_6 for $f(x) = \dfrac{2}{x}$ on the interval $[1, 4]$ with $n = 6$ subintervals.



Now compute M_6.

Problem 3. On the graph below, draw the trapezoid approximation T_6 for $f(x) = \dfrac{2}{x}$ on the interval $[1, 4]$ with $n = 6$ subintervals.



Now compute T_6.

Problem 4. For your computations of M_6 and T_6 in Problems 2 and 3, compute the theoretical error bounds and comment on how each approximation was compared to the "expected error". Is this what you would have predicted?

Problem 5. How large should n be so that the trapezoid approximation T_n of $\int_0^1 xe^{-x}\,dx$ is accurate to within 0.001?

Problem 6. Find an interval $[a, b]$ on which M_n underestimates and R_n overestimates the value of $\int_a^b x^2 \ln(x)\, dx$.

Problem 7. Here are some more integrals to practice.

(a) $\int x \arctan(x)\, dx$

(b) $\displaystyle\int (2x^2+1)e^{x^2}\,dx$

(c) $\displaystyle\int \frac{\ln(x)+1}{x^2}\,dx$

(d) $\displaystyle\int \frac{e^{1/x}}{x^3}\,dx$