Group members:

Warm-up: write the integration by parts (IBP) formula below and explain how it relates to the product rule for differentiation.

Problem 1. Use integration by parts to evaluate the following integrals.

(a) $\int x\cos(5x)\,dx$

(b) $\int x^2 e^{3x}\,dx$

(c) $\int x\sqrt{x+1}\,dx$

Problem 2. (Lecture 2.3, Q10) There are some IBP problems that don't look like IBP problems. Here's an example: $\int \ln(x)\,dx$.

We have to be clever here. I'll give you the u and dv values to get started:

$$u = \ln(x) \qquad \text{and} \qquad dv = 1\,dx.$$

Then the integral we are looking for is $\int \ln(x) \cdot 1\,dx = \int u\,dv$. Now use integration by parts to determine the integral of $\ln(x)$.

Integral of natural log: $\int \ln(x)\,dx =$

You can memorize this integral, but if you forget, now you know how to derive it using integration by parts. Finally, how can you confirm that your answer is an antiderivative of $\ln(x)$?

Some tips for integration by parts:

- In general, you should pick u so that du is *simpler*. For example, choosing $u = x$ in Problem 1a gave us $du = 1$, so the integral on the right side of the IBP formula is much simpler.
- If there is not an obvious choice for u and dv, try setting $dv = 1\,dx$ so that $v = x$. (We did this in Problem 2.) Sometimes this will yield an integral on the right side which you recognize or can solve with another method.
- If you get an integral on the right side that you don't recognize, you may need to *integrate by parts again!*
- When you are integrating a product of two functions (e.g. $\int x \sin(x)\,dx$) and you're not sure which to pick for u, try using the ILATE rule. Remember: this stands for Inverse, Logarithmic, Algebraic (meaning polynomial), Trigonometric, Exponential and the rule says to pick the first type of function on your list as u. The intuition behind this strategy is that as we move down the list I→L→A→T→E, the complexity of the derivative increases.

Problem 3. Try some more examples to practice.

(a) Compute $\int \cos^2(x)\,dx$.

(b) Compute $\int x \ln(x)\,dx$.

(c) Compute $\int x^2 e^x \, dx$.

(d) Compute $\int \sin(x) \ln(\cos(x)) \, dx$.

(e) Compute $\int e^x \sin(x) \, dx$.

Problem 4. Some integrals require a combination of techniques to solve. In the next two problems, use substitutions to simplify the integrals and then solve the new integral using any method.

(a) $\int x^5 \sin(x^3)\, dx$

(b) $\int_1^4 e^{\sqrt{x}}\, dx$

Problem 5. Let A be the area enclosed by $f(x) = \ln(x)$, $x = 2$, and the x-axis.

(a) Use the cross section method to set up an integral computing the volume of the solid obtained by rotating A around the x-axis.

(b) Evaluate the integral you wrote in part (a) to find the volume of the solid.